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Some Significant Developments in Aerodynamics Since 1946

The First Daniel and Florence Guggenheim Memorial Lecture

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IT IS A GREAT HONOR and pleasure for me to give the first Daniel and Florence Guggenheim Memorial Lecture before this distinguished international gathering. The name Guggenheim will be connected forever in the history of aviation with the development of the aeronautical sciences. In fact, the Daniel Guggenheim Fund for the Promotion of Aeronautics, which Daniel Guggenheim established in 1926, gave a great impetus to education and research at a number of American universities. I am personally indebted to the Guggenheim family by the fact that my change of continents from Europe to America originated with Daniel Guggenheim and Robert A. Millikan, world-famous physicist and then head of the California Institute of Technology. The story, as I learned it from R. A. Millikan, was that, in 1926, as the Daniel Guggenheim Fund started to distribute grants to several colleges for the purpose of forming graduate schools for aeronautics, Millikan undertook a trip to Long Island where Mr. Guggenheim lived. During his visit with Mr. Guggenheim, Millikan told him that he would make the greatest mistake of his life if a sizable chunk of the money of the Fund did not go to California. California would be, Millikan insisted, in the near future, the most important center of American aircraft production, due to climatic conditions and availability of space for flying establishments. Mr. Guggenheim answered that he would provide the funds for a graduate school if Millikan would bring to Pasadena from Europe someone familiar with and active in aeronautical research, and especially familiar with the theoretical side of research. In this way I received a call to come to the United States. I was told later that Robert A. Millikan said, "First I aimed at Prandtl—then I settled on Kármán." Prandtl was, without doubt, the leading genius in the early development of modern aerodynamics.

Daniel Guggenheim was a man of great vision and lively interest in progress in aeronautics. I remember that after I went through the United States visiting all the places where Guggenheim funds were invested for the promotion of aeronautical science, he asked me what he should do in addition to supporting the schools, so that aeronautical science would flourish in the United States as it had grown and brought ripe fruits in several European countries during the first two decades of our century. I suggested that it would be extremely desirable to have a kind of Handbook of Aerodynamic Theory that would lay down the present state of fundamental aeronautical knowledge, so that young American scholars would know from which point to start. Then I said jokingly, "In Europe it was very helpful that young scientists could meet in quiet cafés for informal discussions. Unfortunately there is no equivalent institution in the United States." Mr. Guggenheim answered, "All right, I shall provide the money for the book." (As a matter of fact, this was the origin of Durand's "Aerodynamic Theory," a collective work based on international authorship which has had a great influence on the development of American scientific literature in the field of Aerodynamics.) "However," he continued, "I will not go into the cafeteria business—even for science's sake!"

Daniel Guggenheim, born in Philadelphia in 1856, was of the second generation in the leadership of the large Guggenheim enterprises in mining and affiliated industries. His son, our great friend Harry, once told me jokingly that, in his grandfather's time, a really good man, in order to be recognized as such, had to create a great fortune. When his father was in business, the main duty was to broaden the scope and to maintain

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the level of the enterprises. "Of me," he said, "they only expect that I spend the money usefully and gracefully."

I am sure that this formulation of Harry F. Guggenheim's activities is extremely modest. He has an impressive record of public service; he was a successful ambassador of the United States to Cuba. There is no doubt that his interests in aeronautics and his personal experience in aviation, especially as a naval aviator during the first World War, were the decisive factor for Daniel Guggenheim's donations being directed toward the promotion of the aeronautical sciences. Also, the organization of the Safe Aircraft Competition in 1930, the active support of Professor Goddard's early efforts in rocket research, and the establishment of the Daniel and Florence Guggenheim Jet Propulsion Centers at C.I.T. and Princeton University, the Institute of Flight Structures at Columbia University, and the Center for Aviation Health and Safety at Harvard are far beyond the qualification "spend the money usefully and gracefully." They are proofs of an extraordinary vision for the kind of education and research needed for the progress of aviation. And last but not least, this "First International Congress for the Aeronautical Sciences" owes its birth not only to the donation of funds by the Daniel and Florence Guggenheim Foundation, but in the first place to the initiative and desire of Harry F. Guggenheim to spend the money for the organization of an international congress open to scientists and engineers of all the nations of the globe which have an association or group devoted to the aeronautical sciences.

I shall now proceed to the technical part of my lecture.

For my review of progress in aerodynamics I would like to choose as a point of departure my Wright Brothers lecture delivered on December 17, 1946. I wrote at that time:

"I believe we have now arrived at the stage where knowledge of supersonic aerodynamics should be considered by the aeronautical engineer as a necessary prerequisite to his art. This branch of aerodynamics should cease to be a collection of mathematical and half-digested, isolated, experimental results. The aeronautical engineer should start to get the same feeling for the facts of supersonic flight as he acquired in the domain of subsonic velocities by a long process of theoretical study, experimental research, and flight experience."

I have the impression that this goal has generally been achieved. There has been rapid progress in the theory of supersonic wings, bodies, and wing-body combinations exposed to supersonic flow, and this progress has been reported in the scientific literature. Experimental work has been carried out in newly created facilities in governmental and industrial laboratories. Finally, advanced students have had improved training at our aeronautical schools. As a result of these developments, a number of engineering firms now have at their disposal large staffs with the necessary grasp and appreciation of the main features of supersonic aerodynamics. As a matter of fact, some of our supersonic bombers have been designed on the basis of more detailed aerodynamic calculations than was possible in the case of the best subsonic aircraft.

However, the honeymoon was short. Soon, new problems were facing the aeronautical engineer, who nowadays is pleased to call himself a missile engineer or even a space technologist.

The problem of ballistic missiles led us to the range of hypersonic speeds, to speeds which are not only comparable with sound velocity but are greater by an order of magnitude than the velocity of sound. At first sight, the hypersonic range introduces certain simplifications into the flow problem; as a matter of fact, it was pointed out as early as 1931 by P. S. Epstein, in a paper devoted to the problem of the drag of artillery projectiles, that for very large Mach Numbers the classical Newtonian law of air resistance becomes valid. At that time, and even at the time of the memorable Volta Congress for High Speed, held at Rome, in 1935, high Mach Numbers belonged to the realm of academic speculation.

The simplification introduced by very high Mach Numbers is largely overbalanced by the complications due to the high temperatures caused by shock and friction. The production of heat, an annoyance in flight at moderate Mach Numbers, becomes a major problem at hypersonic speeds. Furthermore, and as a new complication, one has to take into account the chemical changes in the air, such as dissociation and recombination. No longer are we dealing with pure aerodynamics, nor aerothermodynamics; fluid mechanics must now be combined not only with thermodynamics, but also with chemistry.

I suggested the term *aerothermochemistry* for the combination of these three disciplines. I had mainly in mind problems related to combustion, like flame theory, the theory of quenching, and the like. Hypersonics has now made it necessary to consider chemical reactions which occur without having been planned by the chemical engineer.

Recently, another combination of various disciplines has attracted considerable attention—fluid mechanics and the theory of the electromagnetic field. Some aspects of the motion of conducting liquids acted upon by electromagnetic forces were investigated several decades ago. I also remember that Albert Einstein gave some thought to a thoroughly practical problem in this field: he proposed the design of a refrigerator in which the coolant, for example a liquid metal, would be kept in circulation by an imposed electromagnetic field. He wanted to avoid the use of machinery which needs lubrication. However, the main interest in "magnetofluidmechanics" arose from celestial problems, such as the structure and the motions of galaxies, wave motions and turbulence in cosmic systems and the like. Questions of space flight and problems related to the possible utilization of thermonuclear reactions led to increasing interest in "plasmadynamics." I believe that a systematic nomenclature and classification of these new

branches of combined fluidmechanics, electromagnetic theory and thermodynamics is still lacking. Nevertheless, I believe that in a review of advances of aerodynamics, they should at least be mentioned.

A detailed list of references is appended hereto. I now want to restrict myself to making some comments on some accomplishments in aerodynamics which, I believe, are most significant. One general remark may precede this. It is remarkable how far one can go with the so-called linear theory of supersonic flow, and how many useful conclusions for aircraft design can be derived from this simplified theory which, after all, is nothing else than "acoustics"—i.e., it is based on the assumption that the flow is composed of a uniform parallel flow and a perturbation flow of small magnitude.

(I) Wing Theory

In a paper published in 1946, A. E. Puckett applied the method of singularities, which I had introduced jointly with N. B. Moore, into the first theory of supersonic flow around axially symmetric bodies. We used supersonic sources distributed along the axis of the body. H. S. Tsien introduced doublets (dipoles) in order to include lift in the theory for the case of a body of revolution with angle of attack. These supersonic sources and doublets are formulated in such a way that their effect is restricted to the interior of a Mach cone, the apex of which is the point of singularity. Thus, the solution is essentially identical with the solution of a time-dependent two-dimensional acoustic problem, where the length coordinate in the flow direction replaces the time coordinate of the acoustic phenomena.

Subsonic wings are mostly calculated by means of the concept of the lifting line. Only in a few cases did it prove possible to solve the problem of the lifting surface. However, for the supersonic case, the situation is more favorable. Especially if both the leading and trailing edges are of the supersonic type—i.e., if the components of the flight speed normal to the edges are greater than sound velocity—the local slope of the wing determines directly the required distribution of sources over the wing plan form. These singularities, together with the condition that in undisturbed flow the pressure is equal to the ambient pressure, completely determine the disturbance potential.

Two complications occur if some portions of the perimeter of the wing plan form are edges of the subsonic type:

For the case of a subsonic leading edge, the flows on the upper and the lower surfaces near the edge are no longer independent. The fluid flows around the edge with subsonic velocity, and one obtains a leading-edge suction which has to be calculated. Furthermore, in the sector enclosed between leading edge and limiting Mach line, the pressures resulting from the upper and lower half spaces must be balanced.

In the case of a subsonic trailing edge the Kutta-Joukowski condition has to be satisfied; in other words, the velocity components normal to the edge on the upper and the lower surface must be equalized. This makes

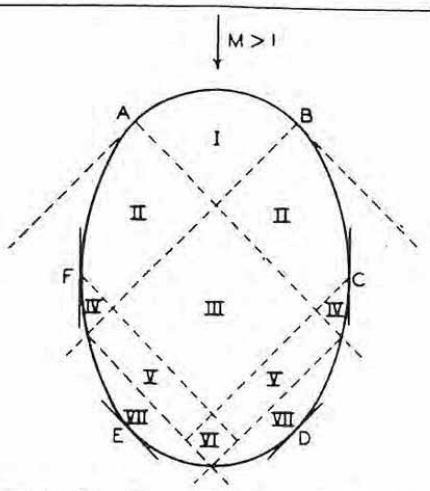


Fig. 1. Plan form of elliptic supersonic wing.

it necessary to compute and compensate the velocities induced by the trailing vortices in certain sectors of the plan form.

Fig. 1 is reproduced from an excellent review of the supersonic wing theory published by R. T. Jones and Doris Cohen in Volume VII of the Princeton series "High Speed Aerodynamics and Jet Propulsion." The Figure shows, for the case of an elliptic plan form, the various domains which have to be treated in different ways. Sector I has a supersonic leading edge and is not influenced by any other portion of the wing plan form. Hence, the Puckett method can be applied directly. Sector II has subsonic leading edges; Sectors IV, V, VI, and VII are evidently influenced by the trailing vortices leaving the subsonic trailing edges $\overline{FE}$ and $\overline{CD}$. The vortex wake can be built up by superposition of horseshoe vortices (Fig. 2). The Figure also shows the domain in which the flow is influenced by such a vortex.

The problem of the partially subsonic leading edge was resolved in an ingenious way by a method apparently independently proposed by J. C. Evvard in the United States and E. A. Krasilshchikova in the USSR.

Before that, H. J. Stewart treated the special case of the delta wing with subsonic straight leading edges by means of the method of conical flows. This method, originally suggested by A. Busemann, became one of the most powerful methods of supersonic aerodynamics. It reduces the problem of three-dimensional flow to a two-dimensional problem which can be solved by means of Laplace's equation.

For example, R. Legendre of France and M. C. Adams, C. E. Brown, W. H. Michael, and R. H. Edwards of the United States treated the case of delta wings with flow separation at the apex as an example of conical flow.

The linearized wing theory made it possible for R. T. Jones to arrive at a series of important results concerning the minimum drag of supersonic wings. He made very ingenious use of the concepts of the reverse flow and the combined flow. In earlier studies I had found that the drag of a thin wing due to thickness remains

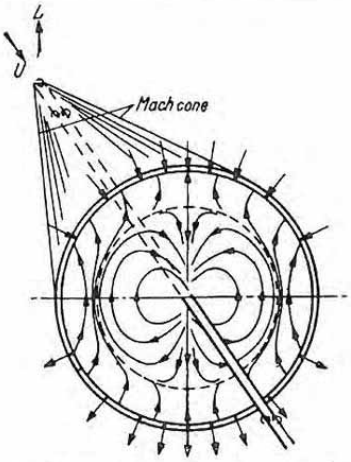


FIG. 2. Horseshoe vortex in supersonic flow.

unchanged if we reverse the flight direction. If we superpose both perturbations, which correspond to the two opposite flight directions, on one parallel uniform flow we arrive at a so-called combined flow. Using this concept and the fact that the drag is independent of the flight direction, R. T. Jones has shown that the drag of a symmetric flat wing of given plan form reaches a minimum if the thickness is distributed over the plan form in such a way that the drag per unit volume is equal for every wing element.

Jones also investigated the condition for the minimum of the drag caused by lift. He found that the optimum lift distribution over a given plan form is such that the downwash is constant over the plan form. It is evident that this result is a kind of generalization of Munk's rule for the optimum distribution of lift along a lifting line, which leads to the elliptical distribution in classical subsonic wing theory.

Many special cases have been investigated from the viewpoint of minimizing drag. Thus, Doris Cohen has shown that, for triangular wings with subsonic edges, the minimum drag due to lift is obtained by using negative spanwise camber over most of the plan form. It is interesting that the idea of conical chamber was used in the design of the wing of the Convair B-58 bomber (also called the Hustler). This development is attributed to C. F. Hall (Ames Laboratory). It is seen from the experimental results represented on Fig. 3 that the conical camber produces leading-edge suction and thus essentially reduces drag.

(II) Slender Wings and Aircraft

With the increase of the speeds of flight, aircraft in general became more and more slender; large aspect ratios disappeared, and present-day aircraft mostly consist of long bodies and wings of small aspect ratios. For such cases, the application of the linearized theory had to be revised. Considerable confusion had been caused by formal application of the Prandtl-Glauert rules for compressibility effects to bodies of revolution and slender wings and bodies in general. This question was clarified by Goethert and others in a satisfactory way. It was shown that the similarity theory of compressible flow which was correct for wings of large aspect ratio gives false results for slender bodies. It was also shown by M. J. Lighthill and G. N. Ward that the computation of the pressure based on the axial component of the disturbance velocity is inexact, since, for example, in the case of axisymmetric bodies, the influence of the radial velocity is often larger. Hence, it became necessary to describe the flow by the appropriate singularities and to compute the pressure including the quadratic terms of the disturbance velocities. In

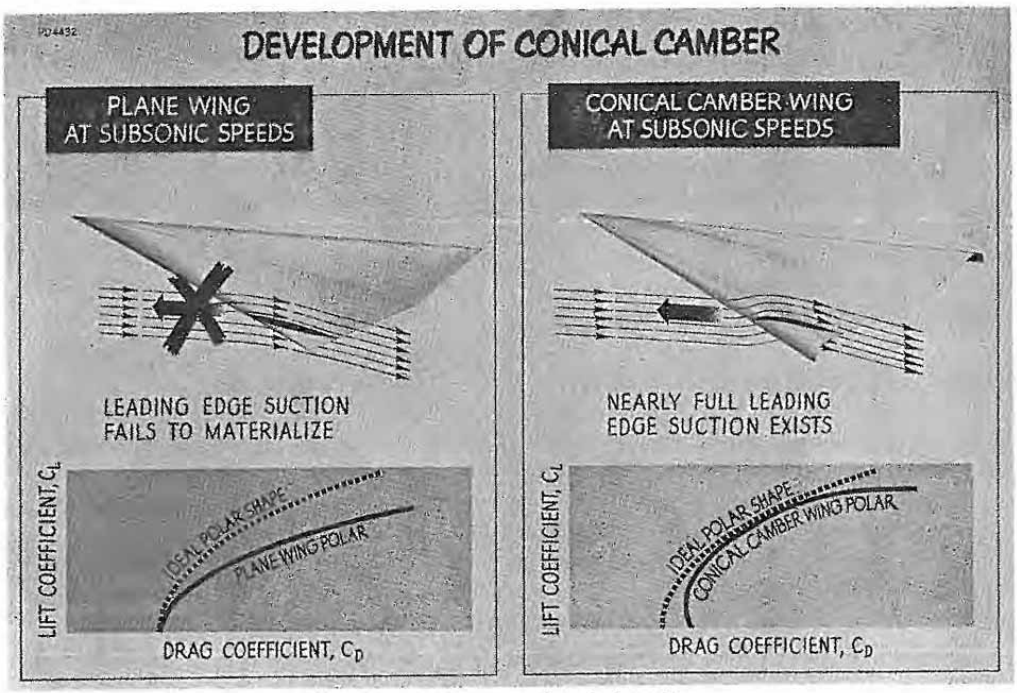


FIG. 3. Development of conical camber.

order to simulate the flow around bodies which are neither thin wings nor axisymmetric bodies, multipoles have to be introduced.

A great simplification was achieved by extension of Munk's classical airship theory to slender bodies in general both for subsonic or supersonic speeds by R. T. Jones. This theory assumes that the flow around a slender body (wing or aircraft) can be considered as a superposition of a parallel uniform flow and a sequence of two-dimensional flows in planes perpendicular to the flight direction as shown in Fig. 4. To some extent this is the opposite of Prandtl's classical assumption for his wing theory. As a matter of fact, for wings of large aspect ratios it was assumed that the flow in planes perpendicular to the wing axis can be identified with a two-dimensional flow with circulation around the wing section.

The slender-body theory in general gives a better approximation to the pressure, forces, and moments acting on the system than the pure linearized theory.

(III) Interference Effects

We shall now consider interference effects between the wing and the body structures of the airplane which are necessary to carry payload, passengers, or fuel. The most significant example of favorable interference is the arrangement popularly known as the *area rule*. W. D. Hayes has shown in his doctoral thesis at the California Institute of Technology that the resulting flow around a system consisting of bodies and thin wings can, at large distances, be represented as originating from singularities distributed along the axis. At Mach Numbers near one the drag is that caused by a single equivalent body of revolution. Now we know—for example, according to the theories of W. Haack and W. R. Sears—the shape of the most favorable bodies of revolution as far as minimum drag is concerned. Thus, the components of the aircraft can be arranged so that, at least for a given Mach number, the equivalent body of revolution approaches the shape corresponding to minimum drag. This principle of equivalence was clearly stated by Hayes and was extended by K. Oswatitsch to the nonlinear case; the extension is important because of the limitations of linearized theory near sonic speed. The principle leads, for example, to the conclusion that in the case of nacelles arranged near the fuselage the additional drag of the nacelles can be compensated by a reduction of the diameter of the fuselage in the appropriate section. We obtain in this way the shape of the fuselage known popularly as the "Marilyn Monroe body." The name "area rule" originates from the fact that for a flow near Mach one the section of the equivalent body of revolution is simply equal to the sum of the areas cut out by a plane laid through the section considered. The agreement of these theoretical ideas with drag measurements was conclusively shown by Richard Whitcomb.

Another class of interference phenomena makes it possible to create favorable interference by reflection of

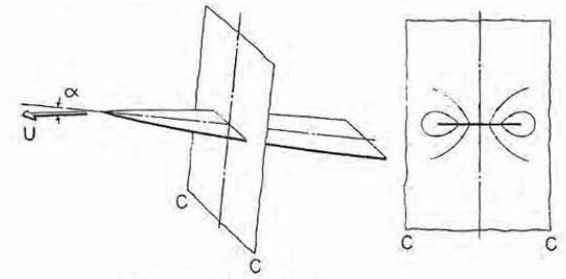


Fig. 4. Wing of low aspect ratio.

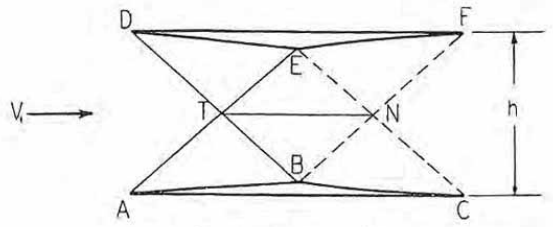


Fig. 5. Busemann biplane.

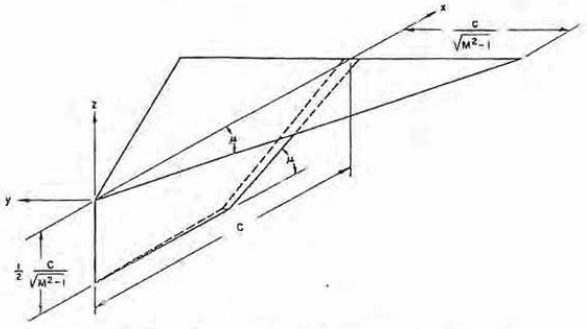


Fig. 6. Wedge-wing lifting configuration.

compression and expansion waves on components of a lifting system.

The oldest example for such a procedure is the so-called Busemann biplane (see Fig. 5). In this case we may transfer pressure from one wing to the rear portion of the other wing (for example, from $\overline{AB}$ to $\overline{EF}$ and from $\overline{DE}$ to $\overline{BC}$), so that the pressures acting on the forward and rear portions of the inner surface are balanced. Thus, the drag due to thickness effect can be eliminated, and we gain volume without additional drag.

An interesting example of similar favorable interference effects is represented by a combination of a horizontal delta wing and a vertical surface of wedge shape arranged below the wing parallel to the flight direction (Fig. 6). One can transfer the pressure created on the surface of the vertical wedge to the lower surface of the wing. Thus the total lift is increased, and one can show that the resulting lift/drag ratio is more favorable than for the case of a simple horizontal wing that would furnish the same lift.

(IV) Aeroelastic Theories

Concerning aeroelastic problems like flutter at high flying speeds, we have to mention the so-called "piston theory" initiated by M. J. Lighthill and worked out by

Holt Ashley and others. This theory assumes that the magnitude of the reaction of a high-speed flow on a wing surface can be approximated at every instant by considering the one-dimensional motion of an air column under the action of a moving piston. It seems that for Mach Numbers superior to $M = 1.5$ this approximate theory furnishes usable results and greatly simplifies the computations.

(V) Nonlinear Theories—Higher-Order Approximations

It is not possible to enter into exact discussion of the highly intricate mathematical problems connected with the development of higher-order approximations for a solution of the equations of supersonic flow even for the cases of plane and axisymmetric motions. However, the main trends of the pertinent approaches will be indicated.

For the purpose of a two-dimensional airfoil theory, Ackeret's fundamental and simple results correspond to a linearized theory. At a relatively early date (1935), Busemann showed the possibility of obtaining a correction of second order in the perturbance velocities, and in 1948 K. Friedrichs provided a means for obtaining shock shapes to the same degree of approximation. Therefore, a rather complete and accurate theoretical treatment of the two-dimensional airfoil problem is available. This applies as long as the effects of entropy changes are relatively unimportant. For practical airfoil shapes the condition seems to be satisfied up to Mach Numbers of 5 or 6. Above this limit, temperature and real-gas effects make the application of ideal-gas laws illusory anyway.

The attempts to develop higher approximations for three-dimensional flows, at least for specific cases, are reviewed by M. J. Lighthill in his article "Higher Approximations" published in Section E, Vol. VI of the Princeton series.

In the linearized theory, simplifications are introduced in three aspects of the problem: first the equation of motion is only approximately correct; second, the boundary conditions at the body are simplified; and third, the pressure is computed on the basis of a simplified relation between pressure and velocity components. The simplest idea is to keep the linearized character of the equation of motion, but to correct the boundary conditions and the pressure-velocity relations. One is tempted to call such theories "hybrid" theories. In general, the procedure leads to results which are in better agreement with exact solutions and/or the experimental evidence, but it can work also in the opposite sense.

J. B. Broderick and M. D. Van Dyke worked out second-order solutions on the basis of this general idea; Van Dyke uses the first-order linearized solutions as a starting point, whereas Broderick starts from the slender-body solution.

One of the main weaknesses of the linearized theory is the fact that it describes the flow quite inexactly at a certain distance from the body or wing In fact, according to the linearized theory the characteristic lines are always straight lines, and their inclination corresponds to the Mach Number of the undisturbed flow. G. B. Whitham has provided a relatively simple procedure for improving the flow-field representation. He assumes that the flow variables predicted by the linearized theory are satisfactorily exact as far as their magnitude is concerned, but must be relocated spatially according to a corrected Mach wave shape. He uses the first-order corrections obtained from the linearized theory for the local sonic velocity and the values for the total velocity obtained from the same theory, and uses these data to construct corresponding Mach waves, including eventual shock surfaces.

The Whitham theory, in which the positions of the Mach waves are changed without changing the values of the flow variables, can be considered as a special case of the general method or "technique" introduced by Lighthill, which he calls a "technique for rendering approximate solutions to physical problems uniformly valid." In this technique—which is based on the methods used by H. Poincaré in his investigations on celestial mechanics—in addition to developing the dependent unknowns in series of successive terms as functions of some parameters, the independent variables or a number of independent variables are also developed as functions of the same parameters. By this trick, in domains where the dependent variables have a singular behavior as functions of the independent variables, they become easy to handle as functions of the new parameter which is used for a series development of the independent variable itself. This technique has proved especially useful in some boundary-layer problems which we shall mention later. As far as applications to nonviscous problems are concerned, the treatment of the mixed flow (hyperbolic and elliptic) about a sharp-edged conical wing by R. Vaglio-Laurin is worth mentioning.

In connection with higher-order approximations, we have to mention also the so-called linearized method of characteristics used by A. Ferri to build up three-dimensional flow fields from nonlinear two-dimensional flow fields which are known by application of classical characteristic methods as used in two-dimensional supersonic aerodynamics since the fundamental works of Prandtl. Ferri finds that the characteristic surfaces of the three-dimensional flow field can be approximated by the envelopes of the characteristic surfaces of the system of two-dimensional flow fields. Hence the analysis of the three-dimensional flow is reduced to that of a flow field with given characteristic surfaces.

(VI) Transonic Flow

The transonic case is characterized by the fact that the difference between flight velocity U and sound velocity c; i.e., the quantity $(U - c)$; is small in comparison with either U or c. It is known that, at least in the case of a two-dimensional flow, for the disturbance po-

tential we arrive at a relatively simple nonlinear equation called Tricomi's equation:

$$(1 - M_\infty^2(\partial^2\varphi/\partial x^2) + (\partial^2\varphi/\partial y^2) = (\gamma + 1)\,(M_\infty/c_\infty)\,(\partial\varphi/\partial x)\,(\partial^2\varphi/\partial x^2)$$

where M_∞ denotes the Mach Number of the main flow, C_∞ the sound velocity at infinity, and γ the ratio of the specific heats. The nonlinear term of this equation contains the product $(\partial\varphi/\partial x)\,(\partial^2\varphi/\partial x^2)$; Oswatitsch and Keune proposed to replace the derivative of the u component of the horizontal velocity, $\partial^2\varphi/\partial x^2$, by a value which is taken as independent of the coordinate y, but may vary stepwise as we proceed along the flow.* They arrived in this way at very useful results for computation of transonic flows around wings and bodies.

To a certain extent, useful results have been achieved for transonic flow calculations by the so-called Kármán-Tsien method, which introduced a "hypothetic fluid" with a simplified pressure-density relation, valid in a limited Mach number range. This idea was further developed and improved by S. Tomotika and K. Tamada.

On the other hand, theoretical studies concerning exact solutions of the Tricomi equation—such as the problem of uniqueness of the solution and the questions: in which cases do we obtain continuous solutions and in which cases do shocks appear?—have not progressed sufficiently in the last ten years.

In an attempt to obtain approximate but useful solutions of transonic flow problems, one replaces the differential equation for the disturbance potential by a system of difference equations. K. Friedrichs and his collaborators used this method; they put the problem into a computing machine. If the problems are properly formulated for the computing device, one can obtain fair approximations even for flows containing discontinuities (shocks) without an elaborate discussion concerning the existence or nonexistence of continuous solutions. Such formulations can also be used in the case of detached shocks in hypersonic flow around blunt bodies.

(VII) Hypersonic Flow

The hypersonic speed range is characterized by the fact that the sonic velocity c is small compared with the flight velocity U. The interest in this speed range was recently very much enhanced by the problems connected with missile design, especially with the design of re-entry vehicles and nose cones.

If we first consider the flow against an inclined surface, for example, the case of a wedge with small apex angle, the theory of inviscid fluids predicts attached and straight shock waves which slightly deflect the flow so that the streamlines practically become parallel to the wedge surfaces. Hence the flow picture is very similar to that assumed in the model treated by Isaac Newton. The essential difference is that according to Newton's model the fluid particles move in parallel straight lines until they hit the surface and then are deflected into a motion along the body surface, whereas according to the ideal-flow theory they are deflected at the shock surface which, however, lies very near to the body surface. Thus, for a small angle of inclination, both for the case of the two-dimensional wedge and that of the axisymmetrical cone, we obtain the result that the pressure acting on the surface element is given by

$$p = (\rho/2)U^2 \cdot 2\sin^2\beta$$

where ρ is the density in the undisturbed fluid, U the velocity of the same, and β the angle of inclination of the surface.

We may call a flow of this nature a Newtonian flow. The actual evidence is, of course, strongly modified by two factors. First, the presence of the solid surface produces viscous effects—i.e., a boundary layer; and second, no mathematically exact sharp edge exists, and, therefore, one always obtains a kind of detached shock with very large curvature near the front portion. The real picture of the flow therefore looks more like the one for the case of a body with a blunt edge, where the "equivalent body" corresponds in a broad sense to the domain occupied by the boundary layer. This description even applies to the flow observed in the case where the undisturbed flow is parallel to one surface of the unsymmetric wedge (see Fig. 7). Furthermore, the large curvature of the detached shock wave introduces vorticity in the inviscid domain of the flow between the shock and the boundary layer.

Nevertheless, the rules of Newtonian flow can be accepted as fair approximations for two- and three-dimensional flows. If the body surface is curved, the flow in the inviscid region is necessarily curved and a correction for the Newtonian law can be obtained by computing the effect of "centrifugal forces" between the shock and the body surface. Such a correction was suggested by A. Busemann at an early date (*Handbuch der Naturwissenschaften*, 1933) and recently further developed by H. R. Ivey and R. R. Morisette, and further by H. R. Ivey, E. B. Klunker, and E. N. Bowen.

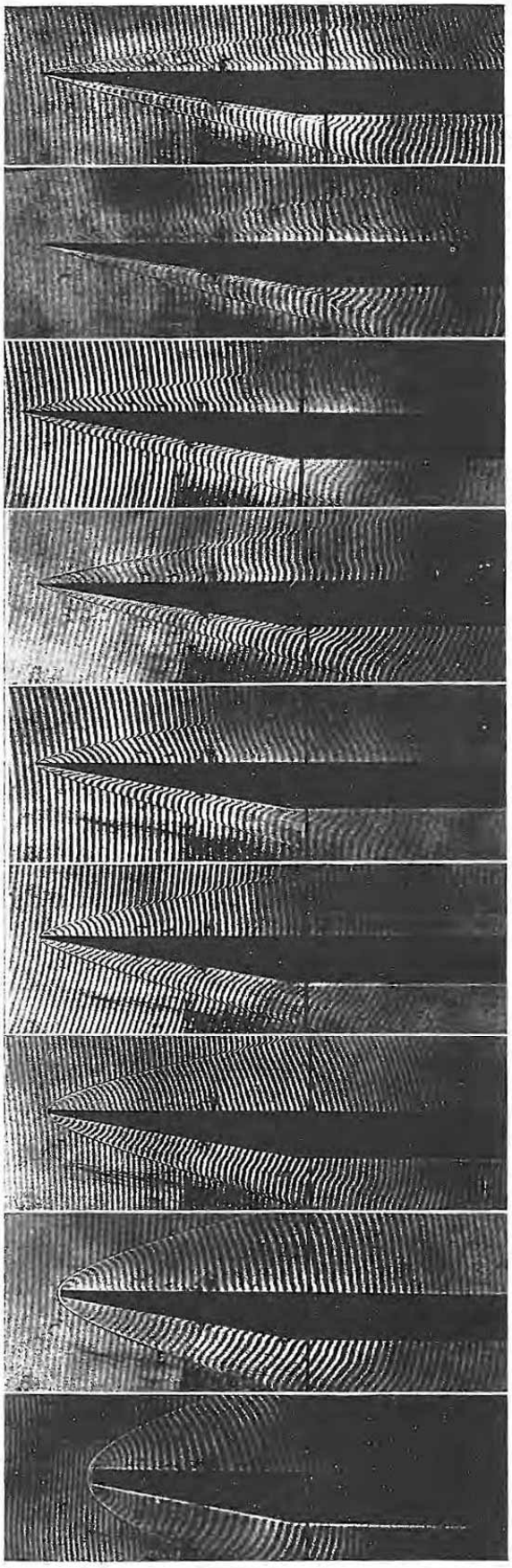
If we consider a blunt body—for example, a cylindrical or spherical body exposed to a flow hitting the body normal to its surface—two considerations have to be introduced.

First, the value of the pressure at the stagnation point is, strictly speaking, unknown, since it is influenced by the thermodynamic process. This was unknown to Newton, as is evident if we consider that he computed the sound velocity, for example, assuming an isothermal compression process.

In fact, there are in addition changes in the distribution of the thermal energy between the several kinds of degrees of freedom in the gas molecules; at the high temperatures produced by the shock we certainly will encounter dissociation and recombination of atoms. Heat-transfer effects will modify the temperature and pressure distribution between the shock and the surface.

The second consideration, which further complicates the problem, is the fact that we have a mixed-flow prob-

* This idea has been developed further by J. R. Spreiter.



lem. Whereas, in the case of the sharp wedge or ogive, the flow in the inviscid region was supersonic on both sides of the shock wave, we now have downstream of the shock a subsonic region followed by a supersonic domain.

Nevertheless, if we consider the general type of flow shown schematically in Fig. 8, we can see that, at least in the neighborhood of the stagnation region, the fate of the individual gas particles is not very different from that assumed in Newton's model. The shock surface approximately follows the body surface and the individual streamlines show that the particles, broadly speaking, are deflected from their initial direction in the direction along the body surface. Correspondingly, we obtain a quasi-Newtonian pressure distribution in the sense that the local pressure coefficient C_p is given approximately by the formula

$$C_p = C_{p\ max} \sin^2 \beta$$

where $C_{p\ max}$ is the local pressure coefficient at the stagnation point and β is the local inclination of the surface relative to the flow direction.

Beyond this quasi-Newtonian pressure distribution, attempts to determine the exact flow conditions in the region between the shock wave and the blunt body were not too successful until recently. There are many uncertainties: first, the location of the shock wave is uncertain; second, the sonic line can be determined only by a kind of iteration process, so that the subsonic and supersonic flow regions really match. Even if we stick to ideal-gas laws, we cannot consider the flow as a potential flow because of the vorticity produced in the shock region. The best approximation is that of constant vorticity along streamlines in the two-dimensional case and constant-vorticity flow in the axially symmetric case. The computation usually requires the help of electronic computers. Among the iteration methods, one used by the Soviet mathematicians Dorodnitsyn and Belotserkovski is reported to be quite successful.

The flow beyond the sonic line can be calculated by the method of characteristics. Recent publications by H. M. Lieberstein and P. R. Garabedian, M. D. Van Dyke, and R. Vaglio-Laurin and A. Ferri deal with this problem. They consider the inverse case—to assume a shock shape and find the corresponding body shape.

Fig. 9 shows experimental pressure measurements for the case of a body consisting of a long circular cylinder with a hemispherical frontpiece. The measurements were made at $M = 7.7$.

Near the stagnation point the pressure distribution corresponds to the quasi-Newtonian rule; on the shoulder, where the curvature of the meridian section changes discontinuously, apparently the rate of the

FIG. 7 (*left*).—Interferograms of flat plate model at various values of the Reynolds Number Re_t based on the leading-edge thickness. $M = 12.7$. Top to bottom: (a) $Re_t = 177$; (b) $Re_t = 578$; (c) $Re_t = 1{,}510$; (d) $Re_t = 2{,}480$; (e) $Re_t = 4{,}050$; (f) $Re_t = 7{,}480$; (g) $Re_t = 15{,}010$; (h) $Re_t = 35{,}200$; (i) $Re_t = 62{,}200$ (Princeton Univ. Dept. of Aero. Engineering, Report No. 326).

pressure drop can be described by the assumption of a Prandtl-Meyer expansion process. Then, however, one would expect that the pressure would rather quickly approach the final value (which because of the expansion in the wake is slightly below the value corresponding to the undisturbed flow). However, observation shows a very slow decrease; evidently this corresponds to an expanding shock-wave surface over the cylindrical portion of the body.

This process was made understandable by application of the analogy between an unsteady motion, in which the flow picture remains similar to itself, and a steady flow, in which the length coordinate in the flight direction replaces the time coordinate of the unsteady process A characteristic example of such unsteady solutions is the well known solution which G. I. Taylor obtained during World War II for the problem of a violent spherical explosion. The propagation of a weak explosion with sound velocity can be considered as one limiting case, namely the case where the energy introduced by the explosion is small relative to the enthalpy of the gas involved. Taylor's problem is the other extreme: he assumed that the energy introduced is very large in comparison with the enthalpy of the gas in which the explosion occurs. However, Taylor's unsteady solution is three-dimensional in space; thus it cannot be used immediately for the description of a steady flow (except perhaps in a four-dimensional space). Recently S. C. Lin solved the problem of a violent cylindrical explosion

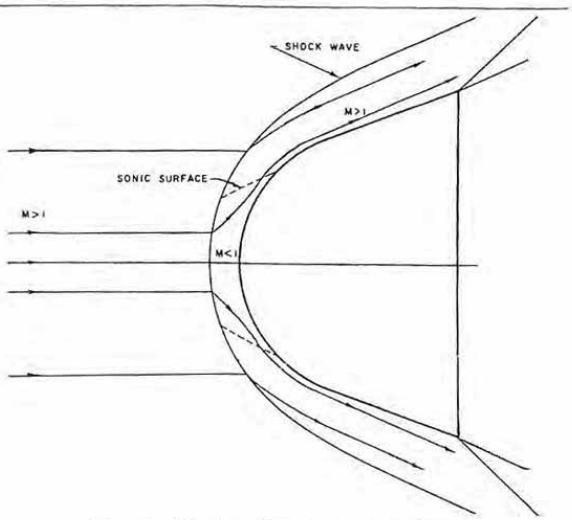


FIG. 8. Blunt body in hypersonic flow.

and this computation led to the "blast-wave theory" of hypersonic motion. Especially, the variable distance between the shock wave and the surface of a long body can be computed by this method, in that one builds up the flow picture in consecutive perpendicular planes of a sequence of solutions for the unsteady cylindrical explosion at various states of the propagation process. The location of the shock wave of the steady flow problem is identical with the location of the expanding wave at the corresponding time element. One assumes that

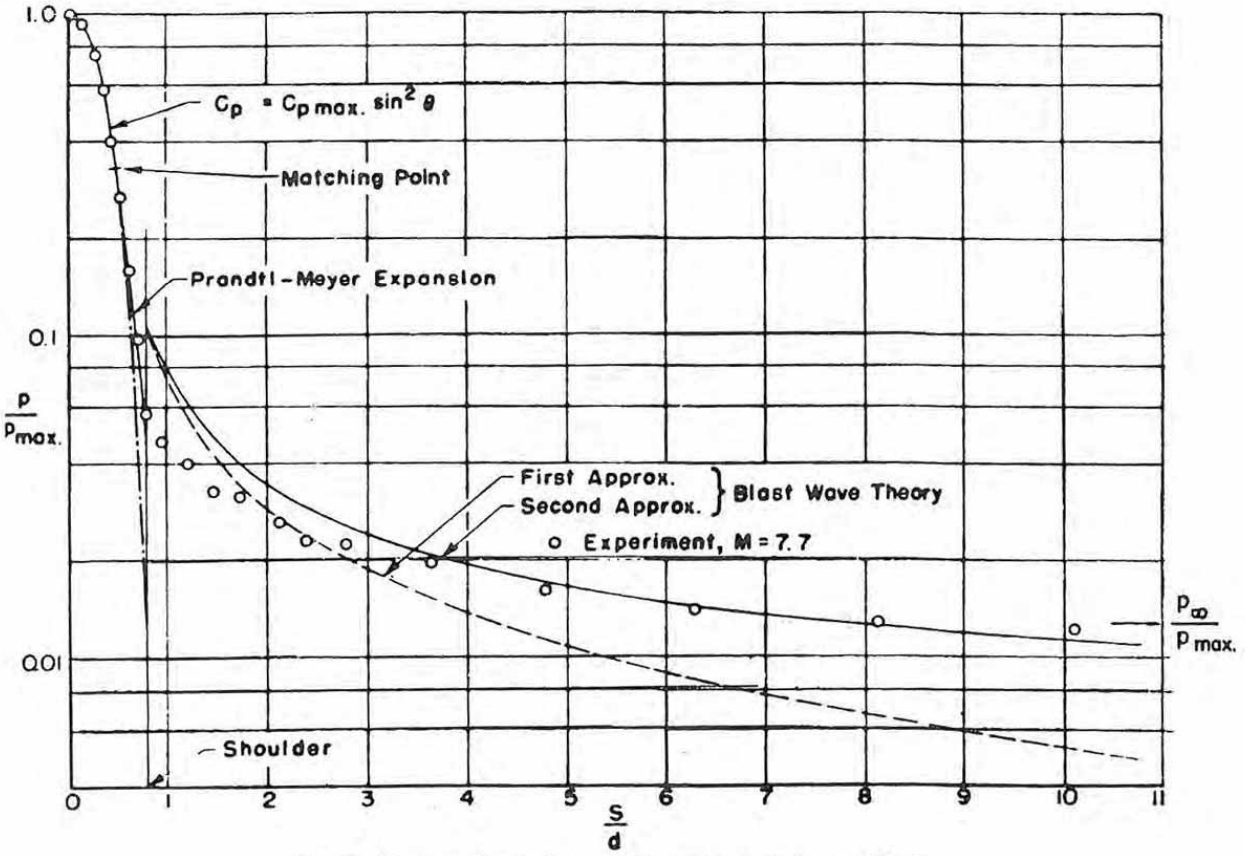


FIG. 9. Pressure distribution on a hemisphere-cylinder combination.

the explosion occurs at the cylindrical body surface and the energy introduced by the explosion corresponds to the energy introduced into the fluid by the resistance of the body; it is, in general, time-dependent. It is seen from Fig. 9 that the results of the blast-wave theory are in fair agreement with the observations. From the rather extensive literature on the subject, the contributions of A. Sakurai, S. C. Lin, L. Lees and T. Kubota, and also H. K. Cheng and A. J. Pallone can be mentioned. Independently, the theory was also developed in Russia, where the self-similar process was named an "auto-model." In particular, L. Sedov and his collaborators worked on the theory of strong explosions. Grodzovski, Chernyi, and Stanynkovich found and employed the analogy between the unsteady and the steady motion.

The true problem of the blunt body exposed to hypersonic flow involves aerothermochemistry, since the temperatures reached behind the shock are so high that the air dissociates. However, before discussing this problem, we want to consider some general questions related to boundary-layer theory.

(VIII) Boundary-Layer Theory

In 1954 we celebrated the fiftieth anniversary of the concept of the boundary layer, in view of the fact that Ludwig Prandtl presented the fundamentals of the boundary-layer theory in 1904 to the International Congress for Mathematics assembled in Heidelberg, Germany. For half a century the new concept proved to be one of the most fruitful ideas in fluid mechanics. The anniversary volume, entitled "Fifty Years of Boundary Layer Research," can give an approximate picture of the main aspect of this development. We also want to mention that, in 1947, Loitsianski published a review of the contributions of Soviet scientists to boundary-layer theory.

Most investigations of boundary-layer problems refer to plane or axisymmetric flows. We will mention later more general three-dimensional cases. At this point we only want to mention that Mangler succeeded, by means of a transformation, in reducing the axisymmetric case to a corresponding two-dimensional problem.

There are two different approaches which were used in the practical solution of boundary-layer problems, especially in the case of incompressible fluids. One is the so-called integral method suggested by me and first used by K. Pohlhausen. This method reduces the problem of finding a solution of a partial differential equation to that of a solution of an ordinary differential equation. Recently, an essential improvement of the method was achieved by I. Tani. The second approach consists also in a reduction of the partial differential equation to an ordinary differential equation by looking for special pressure distributions along the wall which allow similar solutions through the cross sections of the boundary layer. V. M. Falkner, S. W. Skan, D. R. Hartree, and B. Thwaites in the early thirties excelled in the development of such "similarity solutions."

As far as laminar compressible boundary-layer problems are concerned, some special cases were solved before 1946 by A. Busemann, myself, H. S. Tsien, W. Hantsche and H. Wendt, L. Crocco, and by H. W. Emmons and J. G. Brainerd. Special attention was given to the heat-transfer aspect of the problems.

The treatment of the compressible boundary layer in more general cases was greatly facilitated by a clever transformation of the independent variable across the boundary layer, which takes into account the variable density. Such a transformation apparently was first suggested by A. A. Dorodnitsyn and independently discovered by L. Howarth, K. Stewartson, and C. R. Illingworth.

Since this transformation essentially reduces the problem of the compressible boundary layer to the incompressible case, the methods mentioned above could be applied to a broad field of problems. Thus, the integral method could be used for the flow about airfoils and bodies of revolution in supersonic flight. We can mention the contributions of L. E. Kalikhman, H. Weil, P. A. Libby and M. Morduchow, I. E. Beckwith, D. N. Morris and J. W. Smith, and others.

Also, the method of similar solution was combined with the Stewartson-Illingworth transformation by L. Crocco and C. B. Cohen; they also extended the method of Thwaites by putting together sequence of similarity solutions. This procedure was further developed by C. B. Cohen and E. Reshotko. It appears that the work of S. Levy allows a great degree of generality in the formulation of the problem, viz., large temperature changes, including viscous heating and arbitrary values of the Prandtl Number. This nondimensional quantity was taken in many previous investigations to be equal to unity.

For a long time the theory of the laminar boundary layer was considered as having more academic than practical value. Recently, it was found that there are two reasons why the study of the laminar boundary layer is interesting also from the viewpoint of practical applications. First, there is the problem of re-entry of blunt bodies into the atmosphere. The highest value of heat transfer occurs at the stagnation point, and at the beginning the boundary layer probably has laminar structure. Second, the flight of missiles and vehicles at high altitude—i.e., in a medium of extremely low density—enhances the interest for conditions at small Reynolds Numbers.

As we mentioned before, the problem of the boundary layer behind a detached shock produced by hypersonic motion of a blunt body is complicated by changes in the physical and chemical nature of the gas at the high temperatures produced by the shock. The main changes to be expected are changes in the distribution of the thermal energy over the degrees of freedom of the gas molecule, dissociation, and, finally, ionization. The first phenomenon leads especially to a variation in the value of the specific heat and particularly of the ra-

tio between the specific heats; the dissociation may fundamentally change the mechanism of heat transfer by introducing the possibility of diffusion. It is found that ionization does not essentially influence the heat transfer; however, the fact that the gas becomes conductive raises the question of whether there is a practical way to change the structure of the boundary layer by artificially imposed electromagnetic-field effects. In addition, the behavior of an ionized gas against radar waves is of practical interest in view of problems of detection and communication in general.

Concerning the methods for theoretical investigation of the influence of variable physical and chemical characteristics, two limiting cases appear as relatively simple—equilibrium state and frozen state. For example, assume a high degree of dissociation immediately behind the shock and consider this degree of dissociation constant over the flow field; recombination in this case would be restricted to the immediate neighborhood of the body surface. Then, of course, an important question of great practical value arises: whether it is possible to prevent recombination at the surface by so-called noncatalytic surface coating. The application of such materials would essentially diminish heat transfer in the critical region and distribute the heat produced by the shock over a larger region.

The more exact investigation of variable concentration according to some rate law of dissociation and recombination is more complicated, but the calculation can be carried out numerically at least for the stagnation point. The most extensive investigation of stagnation-point conditions may be attributed to J. A. Fay and F. R. Riddell. The most complete discussion of the entire problem of the hypersonic boundary layer is by L. Lees. He found that two assumptions—equilibrium state and Lewis Number (ratio between thermal conductivity and diffusion coefficient) equal to unity—make the computation relatively simple. Thus, the possibility arises of carrying out an exact investigation for the stagnation-point region and of using the ratio between the heat transfer at various points furnished by a simplified theory for an estimate of the heat transfer at other points.

In general it seems that the various assumptions concerning the dissociation process do not change the heat-transfer values more than about 40 to 50 per cent.

Investigations of boundary-layer flow with mass transfer are important in view of the possibilities of cooling by injection of material and also absorption of heat by melting or evaporation of the surface, generally called "ablation."

A remarkable phenomenon was found by experiments on boundary layers in hypersonic flow over a flat plate parallel to the flow direction or inclined to the flow direction at a small angle (sharp wedge). It appears that in such cases the fundamental assumption of the usual theory—that the pressure in the boundary layer is equal to the pressure in the external flow—is quite wrong. One finds considerable pressure increase, apparently induced by the boundary layer itself. We

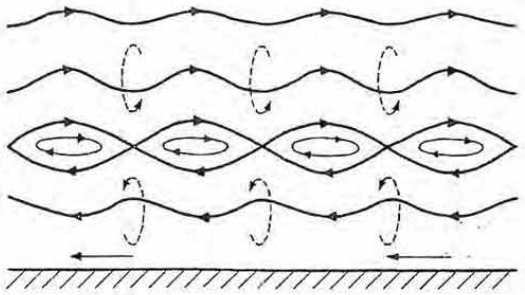


FIG. 10. Illustration to Görtler's theory of transition between laminar and turbulent boundary-layer flow.

mentioned that, probably due to the finite dimension of the leading edge, the flow picture corresponds to a detached shock caused by a kind of equivalent body corresponding to the domain occupied by the boundary layer. Then the motion inside this equivalent body has the general character of a flow through a pipe with increasing cross section, and this kind of motion produces the pressure increase. This phenomenon was called hypersonic boundary-layer - shock-wave interaction.

There is no satisfactory criterion that would permit the prediction of the transition from laminar flow in the boundary layer to turbulent flow—for example, in the case of a blunt body. It is found both theoretically and experimentally that cooling of the surface helps to keep the flow laminar. This point has great practical importance because the heat transfer through a turbulent boundary layer is, in any case, much greater than the transfer through a laminar layer.

Concerning the general problem of stability of the laminar boundary layer, the mathematical theory shows excellent agreement with experiment as far as the damping or the increasing of oscillations is concerned, especially after C. C. Lin corrected Schlichting's original numerical calculations. However, this does not mean that we really understand the mechanism of transition. Recently, Emmons suggested the following mechanism: turbulence is created at isolated spots, and these spots cause contamination of the downstream flow. According to Theodorsen the turbulence originates with the formation of horseshoe vortices which are formed near the walls and penetrate into the fluid. Pfenninger and Lachmann observed similar phenomena in their experiments to keep the boundary-layer flow laminar by means of suction. Görtler proposed the Schlichting-Tollmien waves, as a starting point for the understanding of the transition phenomena, but pointed out that the circulation around the curved streamlines plays a role in the transition, which according to him is an essentially three-dimensional phenomenon (Fig. 10). I. Tani and F. R. Hama have studied the influence of isolated roughnesses on the transition.

Most investigations mentioned above refer to two-dimensional planes of axisymmetrical flow. In recent years, there has been considerable effort devoted to the interesting problems of the boundary-layer behavior when three velocity components are involved. Such

flows arise in a variety of ways; of particular interest for aeronautical applications are the flows over swept wings, over axisymmetric bodies at angle of attack, and in rotating blade systems. Recently, F. K. Moore provided an excellent survey of developments in three-dimensional boundary-layer theory.

The boundary layer on swept wings influences the stalling and lateral control of the wing and has been the subject of considerable research. This problem is idealized by considering the wing to be infinite, so that changes in the spanwise direction are neglected. One can distinguish between a boundary layer normal to the leading edge and a spanwise boundary layer. This consideration leads for incompressible flows to the important "independence principle." According to this, the two boundary layers are independent; one calculates first the chordwise boundary layer in the usual way and then the spanwise boundary layer. The combination of the two boundary layers permits the streamlines within the boundary layer to be constructed. In general, these streamlines differ from those in the external flow so that secondary flows arise. Research on this problem was carried out by L. Prandtl, V. Struminsky, R. T. Jones, and W. R. Sears.

For the compressible case the density depends on both the chordwise and spanwise velocity components, so that the momentum equations in the two directions are "coupled" through the energy equation. The well-known Crocco integral applies in this case under certain restrictions. However, the most general case of chordwise pressure gradient and heat transfer requires simultaneous solution of the chordwise and spanwise momentum equations. Recently E. Reshotko and I. E. Beckwith provided a solution for the boundary-layer and heat-transfer characteristics in the neighborhood of the stagnation line, such as arise at the leading edge of a blunt-nosed, swept wing. This analysis shows that significant reductions in heat transfer can be achieved by sweeping the leading edge of a wing; thus, the concept of sweep is seen to be useful also for hypersonic flight.

(IX) Theory of Turbulence

The theory of turbulence has two separate aspects. Evidently there is some analogy between the random motion of molecules in laminar flow and the random motion of eddylike formations in turbulent flow. The random motion of molecules leads to definite laws of molecular viscosity, heat conduction, and diffusion. In a similar way, the random motion we observe, on a much larger scale, in the turbulent flow of rivers, canals, pipes, and boundary layers apparently also results in definite laws for momentum transfer, energy transfer, and the transfer of matter, which we call turbulent friction, heat transfer, and diffusion, respectively. This analogy between the molecular and the turbulent processes was recognized and treated at a rather early date by Osborne Reynolds. The great difference between the two concepts is the fact that in the case of the molecular random motion the elements are well defined, as molecules, whereas in the case of the turbulent motion they are not given a priori. Hence, whereas in the first case the Boltzmann theory shows the right way to a definite theoretical solution of the problem, in the case of turbulent motion new principles must be found.

Because of this situation, the turbulence theory developed in two essentially different directions. One school of thought endeavored to find half-empirical relations which would lead to definite rules for the prediction of the turbulent friction, heat transfer, and diffusion phenomena. On the other hand, very interesting ideas were proposed for building up a systematic statistical theory of turbulence.

As far as the first line of development is concerned, for the case of an incompressible fluid a satisfactory state of affairs was reached by the introduction of a concept of the mixing length—as a kind of generalization of the mean free path of the kinetic gas theory—due to L. Prandtl, and then, I believe, by the logarithmic law for the velocity distribution, which I found in 1930. Assuming the validity of the logarithmic law for the portion of the flow field which is mainly influenced by the wall, and the so-called "velocity decrement" law for the rest of the field, the turbulent flow in the boundary layer, in a circular pipe, and in a two-dimensional channel can be predicted for given Reynolds Numbers. The application of the integral method used by myself and Pohlhausen, also mentioned in connection with the laminar boundary layer, makes it also possible to calculate general boundary-layer problems. In this direction, F. Clauser recently proposed interesting new concepts and computing methods.

We have to mention that the concept of local similarity of the turbulent flow picture is an important factor as a lead for further development of the theory. When I first presented the idea in 1930, its practical formulation may have been somewhat over-simplified, but I believe—especially after the concept was clarified in some of my own publications together with C. C. Lin and in those of C. C. Lin and his collaborators—that it will have considerable influence on further development of the theory of turbulence.

The main need today is for a reliable prediction of the practically important quantities, especially skin friction and heat transfer for compressible fluids, particularly for supersonic boundary layers at higher Mach Numbers. Fig. 11 convincingly shows the discrepancies between the predictions derived from various proposed "theories."

The diagram, taken from NACA TN 3097 (1954), presents the ratio between the predicted value of the skin-friction coefficient in a compressible boundary layer and the well-known value of the same coefficient for incompressible flow. The experimental value for $M = 4$ is about 0.5. I notice that the value which goes with my name is closer to the experimental value than many others deduced by later elaborate theories. I considered my so-called theory which I proposed more than

twenty years ago as a "guestimate." I simply substituted for the density, in the formula for incompressible flow, the value corresponding to the stagnation temperature which develops at the wall. This procedure naturally underestimates the skin friction, since to be correct some appropriately determined average temperature should be used. Unfortunately, the results of the theories of the Soviet scientists A. A. Dorodnitsyn and L. A. Kalikhman are not included in the comparison.

The systematic development of a statistical theory of turbulence was initiated by G. I. Taylor in 1935. He introduced the concept of isotropic turbulence, which was later modified and somewhat generalized by G. K. Batchelor to homogeneous turbulence. Batchelor's book, published by the Cambridge University Press in 1953, contains extremely valuable information on the developments of the statistical theory. Taylor proposed the average values of velocity correlations as a means of description of the turbulent flow. In 1938, L. Howarth and I gave a general analysis of the correlations and initiated the branch of the statistical theory which can be designated as the "dynamics of turbulence." We arrived at a differential equation for the prediction of the variations of the correlation values with time—i.e., the propagation of turbulence. G. I. Taylor himself introduced the representation of the turbulent fluctuations by spectral functions as an alternative for the correlation functions. J. Kampé de Feriet investigated the concept of the spectral tensor especially from the mathematical point of view. The spectral viewpoint was adopted by Obukoff and Heisenberg in their general investigations. Perhaps the most important result of these developments is a universal law for the spectral function—i.e., turbulent energy vs. wave number—which is valid for high frequencies and was found independently by A. N. Kolmogoroff and W. Heisenberg. S. Chandrasekhar made significant contributions in this direction of the development.

The theory of homogeneous turbulence—which covers the case of a turbulent fluctuation field without mean flow or the turbulent field superposed upon a uniform stream—is in relatively good shape. However, the more interesting and practically more important case, the theory on the mechanism of turbulent shear, is still in its initial phase. From a physical point of view, the main difficulty seems to be that for the momentum transfer the large eddies are mostly responsible, whereas the energy dissipation is performed by the small eddies. This causes a diffusion of energy from low to high frequencies which does not comply with Onsager's general energy-transfer scheme which provides the real basis for Kolmogoroff's and Heisenberg's theories.

From the mathematical point of view, one could expect that the dynamic equation first written up by me, jointly with L. Howarth, would apply also to the general case of nonisotropic turbulence. However, this equation essentially predicts the fate of the second-order and third-order correlations only if the higher correlations

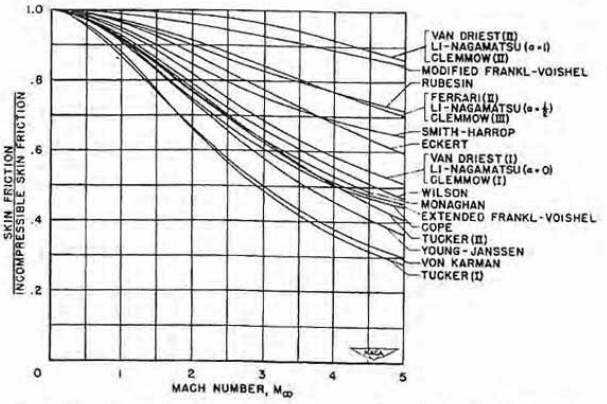


FIG. 11. Results of various analyses of turbulent skin friction for compressible flow over an insulated flat plate (NACA TN 3097).

are known. I believe real progress was made by the hypothesis that a so-called "quasi-normal joint-probability" (Gaussian probability) can be assumed for the velocity field. This assumption makes it possible to express the fourth-order correlations essentially by products of the second-order correlations. It was first suggested by M. Millionschikoff. Attempts were recently made to apply the hypothesis for the solution of the shear problem by J. Proudman, W. H. Reid, T. Talsumi, and A. Craya. Other approaches for the solution of the shear problem were made by S. Chandrasekhar, W. V. R. Malkus, J. M. Burgers, and M. Mitchner. All these theories are in an initial state. We are still far away from a theoretical deduction of the basic laws for shear and heat transfer which would give results in agreement with experimental evidence. I believe that the refinement of the similarity concept has to play a role in the establishment of a final theory.

The experimental evidence on turbulent boundary layer and turbulent shear was very much enriched by the excellent work of many experimenters such as L. Kovasznay, J. Laufer, A. A. Townsend, G. B. Schubauer, and others.

I mentioned before that "heavenly turbulence"—i.e., the turbulent phemonena observed in gaseous clouds of cosmic dimensions—fascinated the imagination of quite a number of mathematicians, astronomers, and aerodynamicists. It was one of the problems which contributed to the interest in the most recent branch of fluid mechanics: "magnetofluidmechanics."

I may close with the remark that it appears to me that even in this so-called nuclear or space age, aerodynamics is not a science to be shelved as an obsolescent branch of the physical sciences. Of course, we may have to study the methods and results of many sister sciences more than in the past; viz., in addition to mathematics, mechanics, and thermodynamics, also chemistry and electromagnetism. However, I hope that the future training of young aerodynamicists will enable them to cope with the problems of the future.

It is my agreeable duty to express my thanks and appreciation to my friends P. A. Libby and C. C. Lin for their assistance in collecting the material treated in

this lecture. Professor Libby collected the papers dealing with supersonic wing theory, slender aircraft, interference effects, higher approximations, hypersonic flow, and boundary-layer theory, and Professor Lin those dealing with the theory of turbulence. The choice of authors mentioned in the paper is somewhat arbitrary. I am also indebted to Prof. Wallace D. Hayes for letting me have the galley proofs of his book on hypersonic flow which he has written jointly with R. F. Probstein, and which will appear in the near future. Prof. I. Tani has prepared abstracts for me of important Japanese papers published in the last decade. The problems of flow in rarefied gases, sometimes referred to as problems of *superaerodynamics* are not treated here at all; the reader may find good summaries in the Preceedings of an International Conference held in Nice, France, in July, 1958. Also, the aeroelastic problems—e.g., flutter at high speed—are only briefly mentioned.

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